

## 10EC55

Fifth Semester B.E. Degree Examination, July/August 2021 Information Theory and Coding

Time: 3 hrs.
Max. Marks:100

## Note: Answer any FIVE full questions.

1 a. Define: i) Self information
ii) Entropy
iii) Additive property
iv) Extremal property.
(04 Marks)
b. Show that the entropy of the following probability distribution is $2-\left(\frac{1}{2}\right)^{n-2}$.

| Symbols : | $\mathrm{x}_{1}$ | $\mathrm{x}_{2}$ | $\mathrm{x}_{3}$ | $\cdots$ | $\mathrm{x}_{\mathrm{n}-1}$ | $\mathrm{x}_{\mathrm{n}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{P}(\mathrm{X}=\mathrm{x})$ | $\frac{1}{2}$ | $\frac{1}{4}$ | $\frac{1}{8}$ | $\cdots$ | $\frac{1}{2^{n-1}}$ | $\frac{1}{2^{\mathrm{n}-1}}$ |

(08 Marks)


Fig. Q1(c)
(08 Marks)
2 a. Design a system to evaluate the heading of a collection of 400 vehicles. The heading levels are Straight(S), Turning left(L), Turning right(R). This information is to be transmitted every second. Construct a model based on the data given below :
i) On an average during a given reporting interyal, 200 vehicles are heading straight, 100 were turning left and remaining were turning right.
ii) Out of 200 vehicles that reported heading straight, 100 of them reported going straight during the next period, 50 of them turning left and remaining turning right during the next period.
iii) Out of 100 vehicles that reported turning during a particular period, 50 of them continued their turn and remaining headed straight during the next period.
iv) The dynamics of the vehicle did not permit to change their heading from left to right or right to left during subsequent periods.
I) Find entropy of each state II) Find system entropy III) Find the rate of transmission assuming symbol rate as one second.
(10 Marks)
b. Consider the statistically independent source whose source alphabet $S=\left\{\mathrm{s}_{0}, \mathrm{~s}_{1}, \mathrm{~s}_{2}, \mathrm{~s}_{3}, \mathrm{~s}_{4}, \mathrm{~s}_{5}\right\}$ with $\mathrm{P}\left\{\frac{1}{2}, \frac{1}{8}, \frac{1}{4}, \frac{1}{32}, \frac{1}{16}, \frac{1}{32}\right\}$. Using Shannon Fano algorithm, find the codewords of all the messages for the first order source. Compute entropy, efficiency and variance. (06 Marks)
c. Define Kraft Mc. Millan inequality property.
d. Write the decision tree for the following sequence $1101011001010110 \ldots$ Use the codeset given $\left\{\mathrm{s}_{0}: 01, \mathrm{~s}_{1}: 10, \mathrm{~s}_{2}: 110, \mathrm{~s}_{3}: 111, \mathrm{~s}_{4}: 1101\right\}$.
(02 Marks)

3 a. Design Huffman Ternary code for the following scheme
$\mathrm{S}=\left\{\mathrm{s}_{1}, \mathrm{~s}_{2}, \mathrm{~s}_{3}, \mathrm{~s}_{4}, \mathrm{~s}_{5}, \mathrm{~s}_{6}\right\}$
$\mathrm{P}=\left\{\frac{1}{3}, \frac{1}{4}, \frac{1}{8}, \frac{1}{8}, \frac{1}{12}, \frac{1}{12}\right\} \quad \mathrm{X}=\{0,1,2\}$.
(06 Marks)
b. For a channel with five inputs and four outputs ; the various matrices are given below : compute various entropies.

$$
\mathrm{X}=\left[\begin{array}{c}
0.25 \\
0.4 \\
0.15 \\
0.15 \\
0.05
\end{array}\right]
$$

$$
\mathrm{P}(\mathrm{X}, \mathrm{Y})=\begin{aligned}
& \mathrm{x}_{1} \\
& \mathrm{x}_{2} \\
& \mathrm{x}_{3} \\
& \mathrm{x}_{4} \\
& \mathrm{x}_{5}
\end{aligned}\left[\begin{array}{cccc}
\mathrm{y}_{1} & \mathrm{y}_{2} & \mathrm{y}_{3} & \mathrm{y}_{4} \\
0.25 & 0 & 0 & 0 \\
0.1 & 0.3 & 0 & 0 \\
0 & 0.05 & 0.1 & 0 \\
0 & 0 & 0.05 & 0.1 \\
0 & 0 & 0.05 & 0
\end{array}\right] .
$$

(08 Marks)
c. What is a Binary Erasure Channel? Obtain an expression for the channel capacity of the Binary Erasure Channel.
(06 Marks)
a. State Shannon $\sim$ Hartley Law. Derive an expression for maximum capacity of a noisy channel.
(06 Marks)
b. Show that for an infinite bandwidth signal energy to noise ratio $\frac{E}{\eta}$ approaches a limiting value.
(06 Marks)
c. What is a deterministic channel? List their properties.
(04 Marks)
d. A Gaussian channel has a bandwidth of 4 KHz and double sided noise power spectral density of $10^{-14}$ watts $/ \mathrm{Hz}$. Signal power is maintained at a level of $\leq 0.1 \mathrm{MW}$. Calculate the channel capacity.
(04 Marks)

5
a. What are the various methods of controlling errors?
(02 Marks)
b. For a systematic $(6,3)$ liner code, the parity matrix $\mathrm{P}=\left[\begin{array}{lll}1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0\end{array}\right]$. Find all code vectors.
(06 Marks)
c. For a systematic $(6,3)$ linear code with $G=\left[\begin{array}{llllll}1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0\end{array}\right]$ and received codevector $R=\left[r_{1}, r_{2}, r_{3}, r_{4}, r_{5}, r_{6}\right]$. Construct the corresponding syndrome calculation circuit. Demonstrate working principles through an examples.
(06 Marks)
d. Design ( $\mathrm{n}, \mathrm{K}$ ) Hamming code with a minimum distance of dmin $=3$ and a message length of 4 bits for a message $\left\{\mathrm{m}_{0}, \mathrm{~m}_{1}, \mathrm{~m}_{2}, \mathrm{~m}_{3}\right\}=\left\{\begin{array}{lll}1 & 1 & 1\end{array} 0\right\}$.
(06 Marks)
6 a. Write the properties of binary cyclic codes and also list their advantages.
(06 Marks)
b. $A(15,5)$ linear cyclic codes has a generator polynomial $g(x)=1+x+x^{2}+x^{4}+x^{5}+x^{8}+x^{10}$.
i) Draw the block diagram of an encoder and syndrome calculator for this code
ii) Find the code polynomial for the message polynomial $\mathrm{D}(\mathrm{x})=1+\mathrm{x}^{2}+\mathrm{x}^{4}$ in systematic form.
iii) Is $V(x)=1+x^{4}+x^{6}+x^{8}+x^{14}$ a code polynomial?
(14 Marks)

7 a. What is a convolution code? Differentiate linear block codes and convolution codes.
(04 Marks)
b. Fig.Q7(b) shows the convolution encoder.

i) Write the impulse response of the encoder
(03 Marks)
ii) Find the output for the message ( 10011 ) using time domain approach
iii) Find the output for the message ( 00011 ) using transform domain approach
iv) Draw the code tree for the encoder.

8 Write short notes on the following :
a. Golay codes
b. Reed Solomon code
c. Shortened cyclic code
d. BCH code.

